2.1 Exercises

The *HM mathSpace*® CD-ROM and *Eduspace*® for this text contain step-by-step solutions to all odd-numbered exercises. They also provide Tutorial Exercises for additional help.

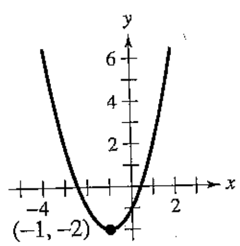
VOCABULARY CHECK: Fill in the blanks.

1. A polynomial function of degree n and leading coefficient a_n is a function of the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdot \cdot \cdot + a_1 x + a_0$ $(a_n \neq 0)$ where n is a ________ ________ and a_1 are ________ numbers.
2. A ________ function is a second-degree polynomial function, and its graph is called a ________.
3. The graph of a quadratic function is symmetric about its ________.
4. If the graph of a quadratic function opens upward, then its leading coefficient is ________ and the vertex of the graph is a ________.
5. If the graph of a quadratic function opens downward, then its leading coefficient is ________ and the vertex of the graph is a ________.

PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at **www.Eduspace.com.**

In Exercises 1–8, match the quadratic function with its graph. [The graphs are labeled (a), (b), (c), (d), (e), (f), (g), and (h).]

(a)



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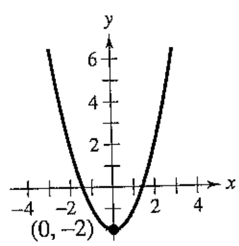

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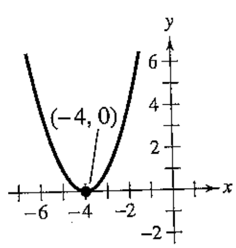

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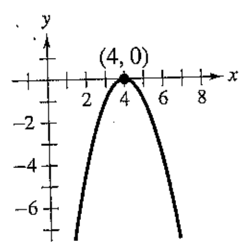

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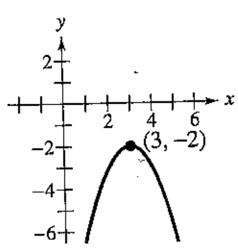

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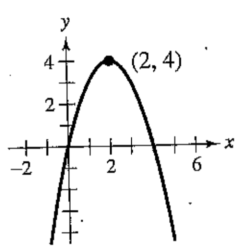

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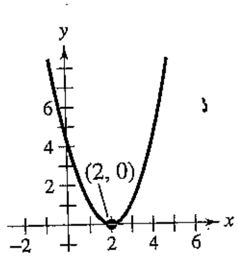

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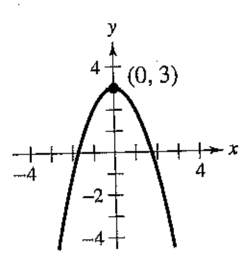


1. $f(x) = (x - 2)^2$
2. $f(x) = (x + 4)^2$
3. $f(x) = x^2 - 2$
4. $f(x) = 3 - x^2$
5. $f(x) = 4 - (x - 2)^2$
6. $f(x) = (x + 1)^2 - 2$
7. $f(x) = -(x - 3)^2 - 2$
8. $f(x) = -(x - 4)^2$

In Exercises 9–12, graph each function. Compare the graph of each function with the graph of $y = x^2$.

9. (a) $f(x) = \frac{1}{2}x^2$ (b) $g(x) = -\frac{1}{8}x^2$
   (c) $h(x) = \frac{3}{2}x^2$ (d) $k(x) = -3x^2$
10. (a) $f(x) = x^2 + 1$ (b) $g(x) = x^2 - 1$
    (c) $h(x) = x^2 + 3$ (d) $k(x) = x^2 - 3$
11. (a) $f(x) = (x - 1)^2$ (b) $g(x) = (3x)^2 + 1$
    (c) $h(x) = \left(\frac{1}{3}x\right)^2 - 3$ (d) $k(x) = (x + 3)^2$
12. (a) $f(x) = -\frac{1}{2}(x - 2)^2 + 1$
    (b) $g(x) = \left[\frac{1}{2}(x - 1)\right]^2 - 3$
    (c) $h(x) = -\frac{1}{2}(x + 2)^2 - 1$
    (d) $k(x) = [2(x + 1)]^2 + 4$

In Exercises 13–28, sketch the graph of the quadratic function without using a graphing utility. Identify the vertex, axis of symmetry, and x-intercept(s).

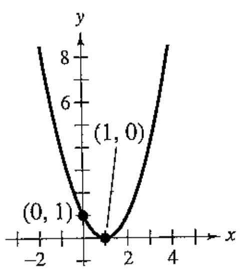
13. $f(x) = x^2 - 5$
14. $h(x) = 25 - x^2$
15. $f(x) = \frac{1}{2}x^2 - 4$
16. $f(x) = 16 - \frac{1}{4}x^2$
17. $f(x) = (x + 5)^2 - 6$
18. $f(x) = (x - 6)^2 + 3$
19. $h(x) = x^2 - 8x + 16$
20. $g(x) = x^2 + 2x + 1$
21. $f(x) = x^2 - x + \frac{5}{4}$
22. $f(x) = x^2 + 3x + \frac{1}{4}$
23. $f(x) = -x^2 + 2x + 5$
24. $f(x) = -x^2 - 4x + 1$
25. $h(x) = 4x^2 - 4x + 21$
26. $f(x) = 2x^2 - x + 1$
27. $f(x) = \frac{1}{4}x^2 - 2x - 12$
28. $f(x) = -\frac{1}{3}x^2 + 3x - 6$

In Exercises 29–36, use a graphing utility to graph the quadratic function. Identify the vertex, axis of symmetry, and x-intercepts. Then check your results algebraically by writing the quadratic function in standard form.

29. $f(x) = -(x^2 + 2x - 3)$
30. $f(x) = -(x^2 + x - 30)$
31. $g(x) = x^2 + 8x + 11$
32. $f(x) = x^2 + 10x + 14$
33. $f(x) = 2x^2 - 16x + 31$
34. $f(x) = -4x^2 + 24x - 41$
35. $g(x) = \frac{1}{2}(x^2 + 4x - 2)$
36. $f(x) = \frac{3}{5}(x^2 + 6x - 5)$

In Exercises 37–42, find the standard form of the quadratic function.

37.



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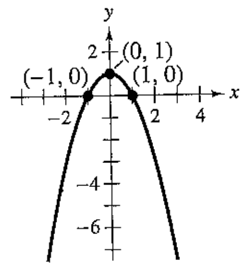

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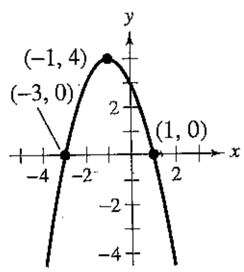

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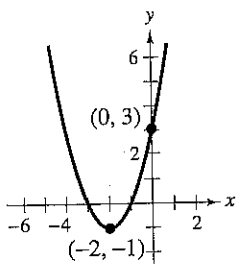

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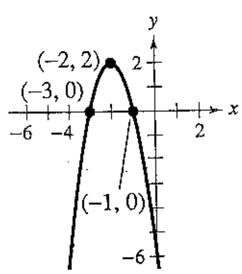

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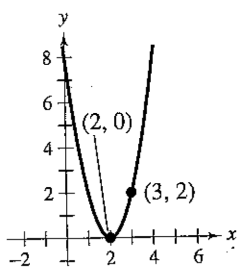
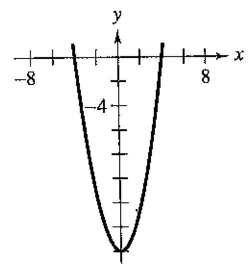


In Exercises 43–52, write the standard form of the equation of the parabola that has the indicated vertex and whose graph passes through the given point.

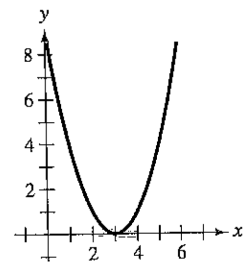
43. Vertex: $(-2, 5)$; point: $(0, 9)$
44. Vertex: $(4, -1)$; point: $(2, 3)$
45. Vertex: $(3, 4)$; point: $(1, 2)$
46. Vertex: $(2, 3)$; point: $(0, 2)$
47. Vertex: $(5, 12)$; point: $(7, 15)$
48. Vertex: $(-2, -2)$; point: $(-1, 0)$
49. Vertex: $\left(-\frac{1}{4}, \frac{3}{2}\right)$; point: $(-2, 0)$
50. Vertex: $\left(\frac{5}{2}, -\frac{3}{4}\right)$; point: $(-2, 4)$
51. Vertex: $\left(-\frac{5}{2}, 0\right)$; point: $\left(-\frac{7}{2}, -\frac{16}{3}\right)$
52. Vertex: $(6, 6)$; point: $\left(\frac{61}{10}, \frac{3}{2}\right)$

Graphical Reasoning **In Exercises 53–56, determine the x-intercept(s) of the graph visually. Then find the x-intercepts algebraically to confirm your results.**

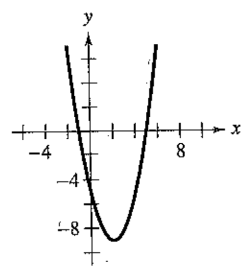
53. $y = x^2 - 16$



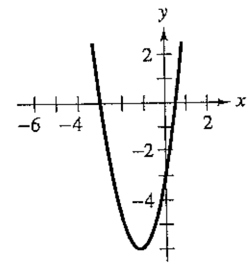
54. $y = x^2 - 6x + 9$



55. $y = x^2 - 4x - 5$



56. $y = 2x^2 + 5x - 3$



In Exercises 57–64, use a graphing utility to graph the quadratic function. Find the x-intercepts of the graph and compare them with the solutions of the corresponding quadratic equation when $f(x) = 0$.

57. $f(x) = x^2 - 4x$
58. $f(x) = -2x^2 + 10x$
59. $f(x) = x^2 - 9x + 18$
60. $f(x) = x^2 - 8x - 20$
61. $f(x) = 2x^2 - 7x - 30$
62. $f(x) = 4x^2 + 25x - 21$
63. $f(x) = -\frac{1}{2}(x^2 - 6x - 7)$
64. $f(x) = \frac{7}{10}(x^2 + 12x - 45)$

In Exercises 65–70, find two quadratic functions, one that opens upward and one that opens downward, whose graphs have the given x-intercepts. (There are many correct answers.)

65. $(-1, 0), (3, 0)$
66. $(-5, 0), (5, 0)$
67. $(0, 0), (10, 0)$
68. $(4, 0), (8, 0)$
69. $(-3, 0), \left(-\frac{1}{2}, 0\right)$
70. $\left(-\frac{5}{2}, 0\right), (2, 0)$

In Exercises 71–74, find two positive real numbers whose product is a maximum.

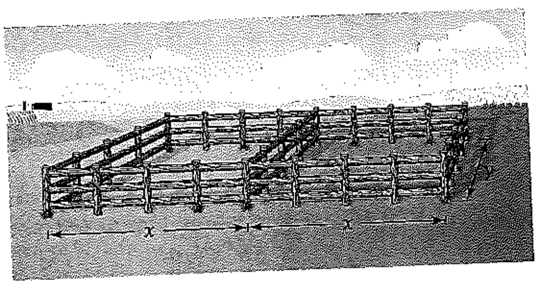
71. The sum is 110.

72. The sum is S.

73. The sum of the first and twice the second is 24.

74. The sum of the first and three times the second is 42.

75. ***Numerical, Graphical, and Analytical Analysis*** A rancher has 200 feet of fencing to enclose two adjacent rectangular corrals (see figure).



(a) Write the area A of the corral as a function of x.

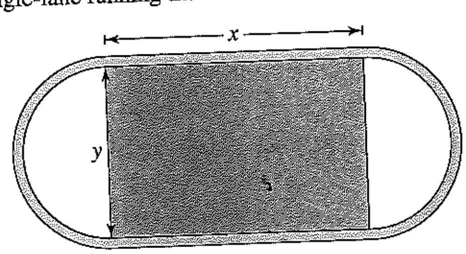
(b) Create a table showing possible values of x and the corresponding areas of the corral. Use the table to estimate the dimensions that will produce the maximum enclosed area.

(c) Use a graphing utility to graph the area function. Use the graph to approximate the dimensions that will produce the maximum enclosed area.

(d) Write the area function in standard form to find analytically the dimensions that will produce the maximum area.

(e) Compare your results from parts (b), (c), and (d).

76. ***Geometry*** An indoor physical fitness room consists of a rectangular region with a semicircle on each end (see figure). The perimeter of the room is to be a 200-meter single-lane running track.



(a) Determine the radius of the semicircular ends of the room. Determine the distance, in terms of y, around the inside edge of the two semicircular parts of the track.

(b) Use the result of part (a) to write an equation, in terms of x and y, for the distance traveled in one lap around the track. Solve for y.

(c) Use the result of part (b) to write the area A of the rectangular region as a function of x. What dimensions will produce a maximum area of the rectangle?

77. ***Path of a Diver*** The path of a diver is given by

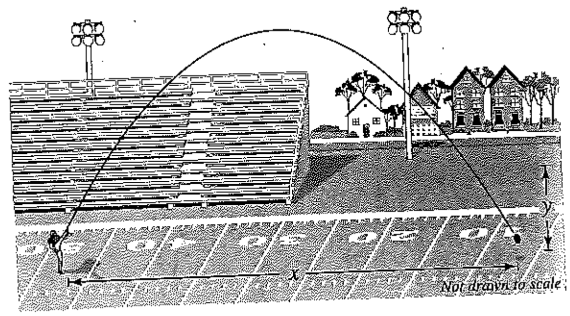
$$y = -\frac{4}{9}x^2 + \frac{24}{9}x + 12$$

where y is the height (in feet) and x is the horizontal distance from the end of the diving board (in feet). What is the maximum height of the diver?

78. ***Height of a Ball*** The height y (in feet) of a punted football is given by

$$y = -\frac{16}{2025}x^2 + \frac{9}{5}x + 1.5$$

where x is the horizontal distance (in feet) from the point at which the ball is punted (see figure).



(a) How high is the ball when it is punted?

(b) What is the maximum height of the punt?

(c) How long is the punt?

79. ***Minimum Cost*** A manufacturer of lighting fixtures has daily production costs of

$$C = 800 - 10x + 0.25x^2$$

where C is the total cost (in dollars) and x is the number of units produced. How many fixtures should be produced each day to yield a minimum cost?

80. ***Minimum Cost*** A textile manufacturer has daily production costs of

$$C = 100{,}000 - 110x + 0.045x^2$$

where C is the total cost (in dollars) and x is the number of units produced. How many units should be produced each day to yield a minimum cost?

81. ***Maximum Profit*** The profit P (in dollars) for a company that produces antivirus and system utilities software is

$$P = -0.0002x^2 + 140x - 250{,}000$$

where x is the number of units sold. What sales level will yield a maximum profit?

82. ***Maximum Profit*** The profit P (in hundreds of dollars) that a company makes depends on the amount x (in hundreds of dollars) the company spends on advertising according to the model

$$P = 230 + 20x - 0.5x^2.$$

What expenditure for advertising will yield a maximum profit?

83. ***Maximum Revenue*** The total revenue R earned (in thousands of dollars) from manufacturing handheld video games is given by

$$R(p) = -25p^2 + 1200p$$

where p is the price per unit (in dollars).

(a) Find the revenue earned for each price per unit given below.

$20

$25

$30

(b) Find the unit price that will yield a maximum revenue. What is the maximum revenue? Explain your results.

84. ***Maximum Revenue*** The total revenue R earned per day (in dollars) from a pet-sitting service is given by

$$R(p) = -12p^2 + 150p$$

where p is the price charged per pet (in dollars).

(a) Find the revenue earned for each price per pet given below.

$4

$6

$8

(b) Find the price that will yield a maximum revenue. What is the maximum revenue? Explain your results.

85. ***Graphical Analysis*** From 1960 to 2003, the per capita consumption C of cigarettes by Americans (age 18 and older) can be modeled by

$$C = 4299 - 1.8t - 1.36t^2, \quad 0 \le t \le 43$$

where t is the year, with $t = 0$ corresponding to 1960. (Source: *Tobacco Outlook Report*)

(a) Use a graphing utility to graph the model.

(b) Use the graph of the model to approximate the maximum average annual consumption. Beginning in 1966, all cigarette packages were required by law to carry a health warning. Do you think the warning had any effect? Explain.

(c) In 2000, the U.S. population (age 18 and over) was 209,128,094. Of those, about 48,308,590 were smokers. What was the average annual cigarette consumption *per smoker* in 2000? What was the average daily cigarette consumption *per smoker*?

Model It

86. ***Data Analysis*** The numbers y (in thousands) of hairdressers and cosmetologists in the United States for the years 1994 through 2002 are shown in the table. (Source: U.S. Bureau of Labor Statistics)

Year	Number of hairdressers and cosmetologists, y
1994	753
1995	750
1996	737
1997	748
1998	763
1999	784
2000	820
2001	854
2002	908

(a) Use a graphing utility to create a scatter plot of the data. Let x represent the year, with $x = 4$ corresponding to 1994.

(b) Use the *regression* feature of a graphing utility to find a quadratic model for the data.

(c) Use a graphing utility to graph the model in the same viewing window as the scatter plot. How well does the model fit the data?

(d) Use the *trace* feature of the graphing utility to approximate the year in which the number of hairdressers and cosmetologists was the least.

(e) Verify your answer to part (d) algebraically.

(f) Use the model to predict the number of hairdressers and cosmetologists in 2008.

87. ***Wind Drag*** The number of horsepower y required to overcome wind drag on an automobile is approximated by

$$y = 0.002s^2 + 0.005s - 0.029, \quad 0 \le s \le 100$$

where s is the speed of the car (in miles per hour).

(a) Use a graphing utility to graph the function.

(b) Graphically estimate the maximum speed of the car if the power required to overcome wind drag is not to exceed 10 horsepower. Verify your estimate algebraically.

2.2 Exercises

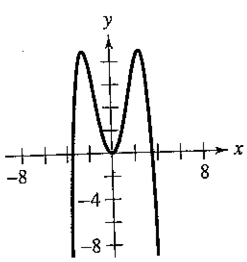
VOCABULARY CHECK: Fill in the blanks.

1. The graphs of all polynomial functions are ________, which means that the graphs have no breaks, holes, or gaps.
2. The ________ ________ ________ is used to determine the left-hand and right-hand behavior of the graph of a polynomial function.
3. A polynomial function of degree n has at most ________ real zeros and at most ________ turning points.
4. If $x = a$ is a zero of a polynomial function f, then the following three statements are true.
   (a) $x = a$ is a ________ of the polynomial equation $f(x) = 0$.
   (b) ________ is a factor of the polynomial $f(x)$.
   (c) $(a, 0)$ is an ________ of the graph f.
5. If a real zero of a polynomial function is of even multiplicity, then the graph of f ________ the x-axis at $x = a$, and if it is of odd multiplicity then the graph of f ________ the x-axis at $x = a$.
6. A polynomial function is written in ________ form if its terms are written in descending order of exponents from left to right.
7. The ________ ________ Theorem states that if f is a polynomial function such that $f(a) \neq f(b)$, then in the interval $[a, b]$, f takes on every value between $f(a)$ and $f(b)$.

PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at **www.Eduspace.com.**

In Exercises 1–8, match the polynomial function with its graph. [The graphs are labeled (a), (b), (c), (d), (e), (f), (g), and (h).]

(a)



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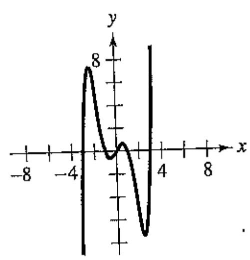

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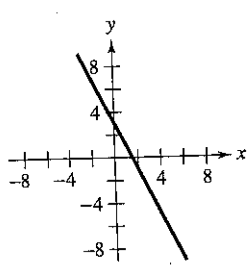

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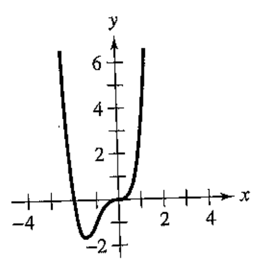

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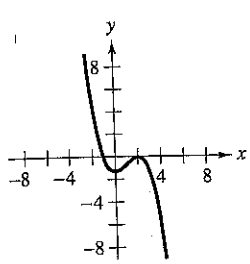

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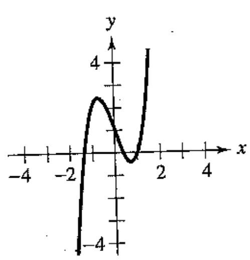

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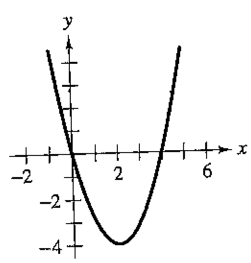

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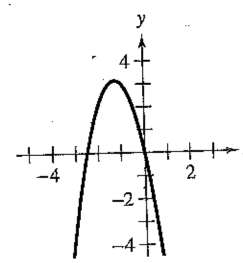


1. $f(x) = -2x + 3$
2. $f(x) = x^2 - 4x$
3. $f(x) = -2x^2 - 5x$
4. $f(x) = 2x^3 - 3x + 1$
5. $f(x) = -\frac{1}{4}x^4 + 3x^2$
6. $f(x) = -\frac{1}{3}x^3 + x^2 - \frac{4}{3}$
7. $f(x) = x^4 + 2x^3$
8. $f(x) = \frac{1}{5}x^5 - 2x^3 + \frac{9}{5}x$

In Exercises 9–12, sketch the graph of $y = x^n$ and each transformation.

9. $y = x^3$
   (a) $f(x) = (x - 2)^3$
   (b) $f(x) = x^3 - 2$
   (c) $f(x) = -\frac{1}{2}x^3$
   (d) $f(x) = (x - 2)^3 - 2$
10. $y = x^5$
   (a) $f(x) = (x + 1)^5$
   (b) $f(x) = x^5 + 1$
   (c) $f(x) = 1 - \frac{1}{2}x^5$
   (d) $f(x) = -\frac{1}{2}(x + 1)^5$
11. $y = x^4$
   (a) $f(x) = (x + 3)^4$
   (b) $f(x) = x^4 - 3$
   (c) $f(x) = 4 - x^4$
   (d) $f(x) = \frac{1}{2}(x - 1)^4$
   (e) $f(x) = (2x)^4 + 1$
   (f) $f(x) = \left(\frac{1}{2}x\right)^4 - 2$

12. $y = x^6$

(a) $f(x) = -\frac{1}{8}x^6$ (b) $f(x) = (x + 2)^6 - 4$

(c) $f(x) = x^6 - 4$ (d) $f(x) = -\frac{1}{4}x^6 + 1$

(e) $f(x) = \left(\frac{1}{4}x\right)^6 - 2$ (f) $f(x) = (2x)^6 - 1$

In Exercises 13–22, describe the right-hand and left-hand behavior of the graph of the polynomial function.

13. $f(x) = \frac{1}{3}x^3 + 5x$ 14. $f(x) = 2x^2 - 3x + 1$

15. $g(x) = 5 - \frac{7}{2}x - 3x^2$ 16. $h(x) = 1 - x^6$

17. $f(x) = -2.1x^5 + 4x^3 - 2$

18. $f(x) = 2x^5 - 5x + 7.5$

19. $f(x) = 6 - 2x + 4x^2 - 5x^3$

20. $f(x) = \dfrac{3x^4 - 2x + 5}{4}$

21. $h(t) = -\frac{2}{3}(t^2 - 5t + 3)$

22. $f(s) = -\frac{7}{8}(s^3 + 5s^2 - 7s + 1)$

Graphical Analysis **In Exercises 23–26, use a graphing utility to graph the functions f and g in the same viewing window. Zoom out sufficiently far to show that the right-hand and left-hand behaviors of f and g appear identical.**

23. $f(x) = 3x^3 - 9x + 1, \quad g(x) = 3x^3$

24. $f(x) = -\frac{1}{3}(x^3 - 3x + 2), \quad g(x) = -\frac{1}{3}x^3$

25. $f(x) = -(x^4 - 4x^3 + 16x), \quad g(x) = -x^4$

26. $f(x) = 3x^4 - 6x^2, \quad g(x) = 3x^4$

In Exercises 27–42, (a) find all the real zeros of the polynomial function, (b) determine the multiplicity of each zero and the number of turning points of the graph of the function, and (c) use a graphing utility to graph the function and verify your answers.

27. $f(x) = x^2 - 25$ 28. $f(x) = 49 - x^2$

29. $h(t) = t^2 - 6t + 9$ 30. $f(x) = x^2 + 10x + 25$

31. $f(x) = \frac{1}{3}x^2 + \frac{1}{3}x - \frac{2}{3}$

32. $f(x) = \frac{1}{2}x^2 + \frac{5}{2}x - \frac{3}{2}$

33. $f(x) = 3x^3 - 12x^2 + 3x$

34. $g(x) = 5x(x^2 - 2x - 1)$

35. $f(t) = t^3 - 4t^2 + 4t$

36. $f(x) = x^4 - x^3 - 20x^2$

37. $g(t) = t^5 - 6t^3 + 9t$

38. $f(x) = x^5 + x^3 - 6x$

39. $f(x) = 5x^4 + 15x^2 + 10$

40. $f(x) = 2x^4 - 2x^2 - 40$

41. $g(x) = x^3 + 3x^2 - 4x - 12$

42. $f(x) = x^3 - 4x^2 - 25x + 100$

Graphical Analysis **In Exercises 43–46, (a) use a graphing utility to graph the function, (b) use the graph to approximate any x-intercepts of the graph, (c) set $y = 0$ and solve the resulting equation, and (d) compare the results of part (c) with any x-intercepts of the graph.**

43. $y = 4x^3 - 20x^2 + 25x$

44. $y = 4x^3 + 4x^2 - 8x + 8$

45. $y = x^5 - 5x^3 + 4x$

46. $y = \frac{1}{4}x^3(x^2 - 9)$

In Exercises 47–56, find a polynomial function that has the given zeros. (There are many correct answers.)

47. 0, 10 48. 0, -3

49. 2, -6 50. -4, 5

51. 0, -2, -3 52. 0, 2, 5

53. 4, -3, 3, 0 54. $-2, -1, 0, 1, 2$

55. $1 + \sqrt{3}, 1 - \sqrt{3}$ 56. $2, 4 + \sqrt{5}, 4 - \sqrt{5}$

In Exercises 57–66, find a polynomial of degree n that has the given zero(s). (There are many correct answers.)

	Zero(s)	*Degree*
57.	$x = -2$	$n = 2$
58.	$x = -8, -4$	$n = 2$
59.	$x = -3, 0, 1$	$n = 3$
60.	$x = -2, 4, 7$	$n = 3$
61.	$x = 0, \sqrt{3}, -\sqrt{3}$	$n = 3$
62.	$x = 9$	$n = 3$
63.	$x = -5, 1, 2$	$n = 4$
64.	$x = -4, -1, 3, 6$	$n = 4$
65.	$x = 0, -4$	$n = 5$
66.	$x = -3, 1, 5, 6$	$n = 5$

In Exercises 67–80, sketch the graph of the function by (a) applying the Leading Coefficient Test, (b) finding the zeros of the polynomial, (c) plotting sufficient solution points, and (d) drawing a continuous curve through the points.

67. $f(x) = x^3 - 9x$ 68. $g(x) = x^4 - 4x^2$

69. $f(t) = \frac{1}{4}(t^2 - 2t + 15)$

70. $g(x) = -x^2 + 10x - 16$

71. $f(x) = x^3 - 3x^2$ 72. $f(x) = 1 - x^3$

73. $f(x) = 3x^3 - 15x^2 + 18x$

74. $f(x) = -4x^3 + 4x^2 + 15x$

75. $f(x) = -5x^2 - x^3$ 76. $f(x) = -48x^2 + 3x^4$

77. $f(x) = x^2(x - 4)$ 78. $h(x) = \frac{1}{3}x^3(x - 4)^2$

79. $g(t) = -\frac{1}{4}(t - 2)^2(t + 2)^2$

80. $g(x) = \frac{1}{10}(x + 1)^2(x - 3)^3$

In Exercises 81–84, use a graphing utility to graph the function. Use the *zero* or *root* feature to approximate the real zeros of the function. Then determine the multiplicity of each zero.

81. $f(x) = x^3 - 4x$

82. $f(x) = \frac{1}{4}x^4 - 2x^2$

83. $g(x) = \frac{1}{5}(x + 1)^2(x - 3)(2x - 9)$

84. $h(x) = \frac{1}{5}(x + 2)^2(3x - 5)^2$

In Exercises 85–88, use the Intermediate Value Theorem and the *table* feature of a graphing utility to find intervals one unit in length in which the polynomial function is guaranteed to have a zero. Adjust the table to approximate the zeros of the function. Use the *zero* or *root* feature of a graphing utility to verify your results.

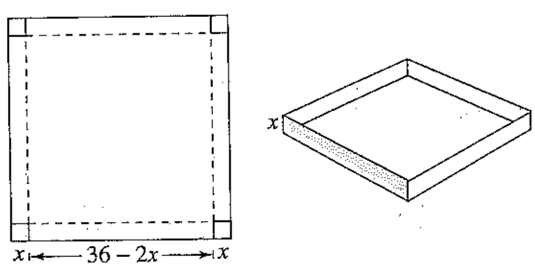
85. $f(x) = x^3 - 3x^2 + 3$

86. $f(x) = 0.11x^3 - 2.07x^2 + 9.81x - 6.88$

87. $g(x) = 3x^4 + 4x^3 - 3$

88. $h(x) = x^4 - 10x^2 + 3$

89. ***Numerical and Graphical Analysis*** An open box is to be made from a square piece of material, 36 inches on a side, by cutting equal squares with sides of length x from the corners and turning up the sides (see figure).



(a) Verify that the volume of the box is given by the function

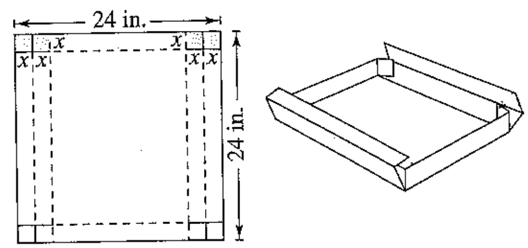
$$V(x) = x(36 - 2x)^2.$$

(b) Determine the domain of the function.

(c) Use a graphing utility to create a table that shows the box height x and the corresponding volumes V. Use the table to estimate the dimensions that will produce a maximum volume.

(d) Use a graphing utility to graph V and use the graph to estimate the value of x for which $V(x)$ is maximum. Compare your result with that of part (c).

90. ***Maximum Volume*** An open box with locking tabs is to be made from a square piece of material 24 inches on a side. This is to be done by cutting equal squares from the corners and folding along the dashed lines shown in the figure.



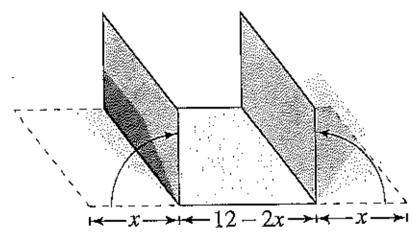
(a) Verify that the volume of the box is given by the function

$$V(x) = 8x(6 - x)(12 - x).$$

(b) Determine the domain of the function V.

(c) Sketch a graph of the function and estimate the value of x for which $V(x)$ is maximum.

91. ***Construction*** A roofing contractor is fabricating gutters from 12-inch aluminum sheeting. The contractor plans to use an aluminum siding folding press to create the gutter by creasing equal lengths for the sidewalls (see figure).



(a) Let x represent the height of the sidewall of the gutter. Write a function A that represents the cross-sectional area of the gutter.

(b) The length of the aluminum sheeting is 16 feet. Write a function V that represents the volume of one run of gutter in terms of x.

(c) Determine the domain of the function in part (b).

(d) Use a graphing utility to create a table that shows the sidewall height x and the corresponding volumes V. Use the table to estimate the dimensions that will produce a maximum volume.

(e) Use a graphing utility to graph V. Use the graph to estimate the value of x for which $V(x)$ is a maximum. Compare your result with that of part (d).

(f) Would the value of x change if the aluminum sheeting were of different lengths? Explain.

Synthesis

True or False? **In Exercises 99–101, determine whether the statement is true or false. Justify your answer.**

99. A fifth-degree polynomial can have five turning points in its graph.

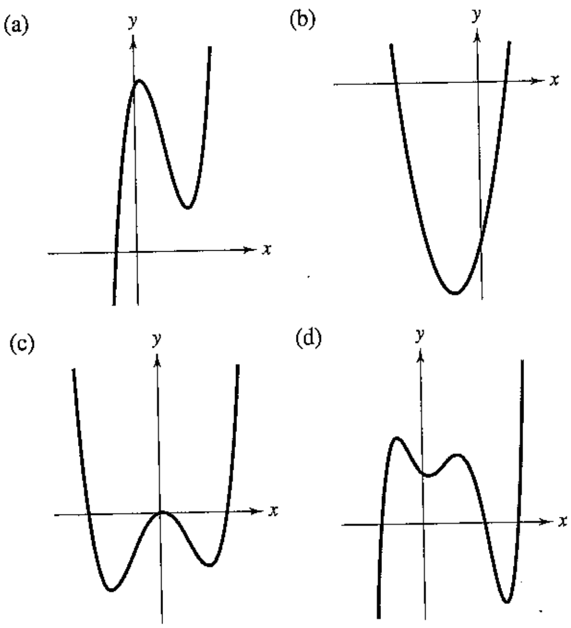
100. It is possible for a sixth-degree polynomial to have only one solution.

101. The graph of the function given by

$f(x) = 2 + x - x^2 + x^3 - x^4 + x^5 + x^6 - x^7$

rises to the left and falls to the right.

102. ***Graphical Analysis*** For each graph, describe a polynomial function that could represent the graph. (Indicate the degree of the function and the sign of its leading coefficient.)



103. ***Graphical Reasoning*** Sketch a graph of the function given by $f(x) = x^4$. Explain how the graph of each function g differs (if it does) from the graph of each function f. Determine whether g is odd, even, or neither.

(a) $g(x) = f(x) + 2$

(b) $g(x) = f(x + 2)$

(c) $g(x) = f(-x)$

(d) $g(x) = -f(x)$

(e) $g(x) = f(\frac{1}{2}x)$

(f) $g(x) = \frac{1}{2}f(x)$

(g) $g(x) = f(x^{3/4})$

(h) $g(x) = (f \circ f)(x)$

104. ***Exploration*** Explore the transformations of the form $g(x) = a(x - h)^5 + k$.

(a) Use a graphing utility to graph the functions given by

$$y_1 = -\frac{1}{3}(x - 2)^5 + 1$$

and

$$y_2 = \frac{3}{5}(x + 2)^5 - 3.$$

Determine whether the graphs are increasing or decreasing. Explain.

(b) Will the graph of g always be increasing or decreasing? If so, is this behavior determined by a, h, or k? Explain.

(c) Use a graphing utility to graph the function given by

$H(x) = x^5 - 3x^3 + 2x + 1.$

Use the graph and the result of part (b) to determine whether H can be written in the form $H(x) = a(x - h)^5 + k$. Explain.

Skills Review

In Exercises 105–108, factor the expression completely.

105. $5x^2 + 7x - 24$

106. $6x^3 - 61x^2 + 10x$

107. $4x^4 - 7x^3 - 15x^2$

108. $y^3 + 216$

In Exercises 109–112, solve the equation by factoring.

109. $2x^2 - x - 28 = 0$

110. $3x^2 - 22x - 16 = 0$

111. $12x^2 + 11x - 5 = 0$

112. $x^2 + 24x + 144 = 0$

In Exercises 113–116, solve the equation by completing the square.

113. $x^2 - 2x - 21 = 0$

114. $x^2 - 8x + 2 = 0$

115. $2x^2 + 5x - 20 = 0$

116. $3x^2 + 4x - 9 = 0$

In Exercises 117–122, describe the transformation from a common function that occurs in $f(x)$. Then sketch its graph.

117. $f(x) = (x + 4)^2$

118. $f(x) = 3 - x^2$

119. $f(x) = \sqrt{x + 1} - 5$

120. $f(x) = 7 - \sqrt{x - 6}$

121. $f(x) = 2[\![x]\!] + 9$

122. $f(x) = 10 - \frac{1}{3}[\![x + 3]\!]$

2.3 Exercises

VOCABULARY CHECK:

1. Two forms of the Division Algorithm are shown below. Identify and label each term or function.

$$f(x) = d(x)q(x) + r(x) \qquad \frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$$

In Exercises 2–5, fill in the blanks.

2. The rational expression $p(x)/q(x)$ is called ________ if the degree of the numerator is greater than or equal to that of the denominator, and is called ________ if the degree of the numerator is less than that of the denominator.
3. An alternative method to long division of polynomials is called ________ ________, in which the divisor must be of the form $x - k$.
4. The ________ Theorem states that a polynomial $f(x)$ has a factor $(x - k)$ if and only if $f(k) = 0$.
5. The ________ Theorem states that if a polynomial $f(x)$ is divided by $x - k$, the remainder is $r = f(k)$.

PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at **www.Eduspace.com.**

Analytical Analysis **In Exercises 1 and 2, use long division to verify that $y_1 = y_2$.**

1. $y_1 = \dfrac{x^2}{x+2}, \quad y_2 = x - 2 + \dfrac{4}{x+2}$
2. $y_1 = \dfrac{x^4 - 3x^2 - 1}{x^2 + 5}, \quad y_2 = x^2 - 8 + \dfrac{39}{x^2 + 5}$

Graphical Analysis **In Exercises 3 and 4, (a) use a graphing utility to graph the two equations in the same viewing window, (b) use the graphs to verify that the expressions are equivalent, and (c) use long division to verify the results algebraically.**

3. $y_1 = \dfrac{x^5 - 3x^3}{x^2 + 1}, \quad y_2 = x^3 - 4x + \dfrac{4x}{x^2 + 1}$
4. $y_1 = \dfrac{x^3 - 2x^2 + 5}{x^2 + x + 1}, \quad y_2 = x - 3 + \dfrac{2(x+4)}{x^2 + x + 1}$

In Exercises 5–18, use long division to divide.

5. $(2x^2 + 10x + 12) \div (x + 3)$
6. $(5x^2 - 17x - 12) \div (x - 4)$
7. $(4x^3 - 7x^2 - 11x + 5) \div (4x + 5)$
8. $(6x^3 - 16x^2 + 17x - 6) \div (3x - 2)$
9. $(x^4 + 5x^3 + 6x^2 - x - 2) \div (x + 2)$
10. $(x^3 + 4x^2 - 3x - 12) \div (x - 3)$
11. $(7x + 3) \div (x + 2)$
12. $(8x - 5) \div (2x + 1)$
13. $(6x^3 + 10x^2 + x + 8) \div (2x^2 + 1)$
14. $(x^3 - 9) \div (x^2 + 1)$
15. $(x^4 + 3x^2 + 1) \div (x^2 - 2x + 3)$
16. $(x^5 + 7) \div (x^3 - 1)$
17. $\dfrac{x^4}{(x-1)^3}$
18. $\dfrac{2x^3 - 4x^2 - 15x + 5}{(x-1)^2}$

In Exercises 19–36, use synthetic division to divide.

19. $(3x^3 - 17x^2 + 15x - 25) \div (x - 5)$
20. $(5x^3 + 18x^2 + 7x - 6) \div (x + 3)$
21. $(4x^3 - 9x + 8x^2 - 18) \div (x + 2)$
22. $(9x^3 - 16x - 18x^2 + 32) \div (x - 2)$
23. $(-x^3 + 75x - 250) \div (x + 10)$
24. $(3x^3 - 16x^2 - 72) \div (x - 6)$
25. $(5x^3 - 6x^2 + 8) \div (x - 4)$
26. $(5x^3 + 6x + 8) \div (x + 2)$
27. $\dfrac{10x^4 - 50x^3 - 800}{x - 6}$
28. $\dfrac{x^5 - 13x^4 - 120x + 80}{x + 3}$
29. $\dfrac{x^3 + 512}{x + 8}$
30. $\dfrac{x^3 - 729}{x - 9}$
31. $\dfrac{-3x^4}{x - 2}$
32. $\dfrac{-3x^4}{x + 2}$
33. $\dfrac{180x - x^4}{x - 6}$
34. $\dfrac{5 - 3x + 2x^2 - x^3}{x + 1}$
35. $\dfrac{4x^3 + 16x^2 - 23x - 15}{x + \frac{1}{2}}$
36. $\dfrac{3x^3 - 4x^2 + 5}{x - \frac{3}{2}}$

In Exercises 37–44, write the function in the form $f(x) = (x - k)q(x) + r$ for the given value of k, and demonstrate that $f(k) = r$.

	Function	*Value of k*
37.	$f(x) = x^3 - x^2 - 14x + 11$	$k = 4$
38.	$f(x) = x^3 - 5x^2 - 11x + 8$	$k = -2$

	Function	*Value of k*
39.	$f(x) = 15x^4 + 10x^3 - 6x^2 + 14$	$k = -\frac{2}{3}$
40.	$f(x) = 10x^3 - 22x^2 - 3x + 4$	$k = \frac{1}{5}$
41.	$f(x) = x^3 + 3x^2 - 2x - 14$	$k = \sqrt{2}$
42.	$f(x) = x^3 + 2x^2 - 5x - 4$	$k = -\sqrt{5}$
43.	$f(x) = -4x^3 + 6x^2 + 12x + 4$	$k = 1 - \sqrt{3}$
44.	$f(x) = -3x^3 + 8x^2 + 10x - 8$	$k = 2 + \sqrt{2}$

In Exercises 45–48, use synthetic division to find each function value. Verify your answers using another method.

45. $f(x) = 4x^3 - 13x + 10$
    (a) $f(1)$ (b) $f(-2)$ (c) $f(\frac{1}{2})$ (d) $f(8)$
46. $g(x) = x^6 - 4x^4 + 3x^2 + 2$
    (a) $g(2)$ (b) $g(-4)$ (c) $g(3)$ (d) $g(-1)$
47. $h(x) = 3x^3 + 5x^2 - 10x + 1$
    (a) $h(3)$ (b) $h(\frac{1}{3})$ (c) $h(-2)$ (d) $h(-5)$
48. $f(x) = 0.4x^4 - 1.6x^3 + 0.7x^2 - 2$
    (a) $f(1)$ (b) $f(-2)$ (c) $f(5)$ (d) $f(-10)$

In Exercises 49–56, use synthetic division to show that *x* is a solution of the third-degree polynomial equation, and use the result to factor the polynomial completely. List all real solutions of the equation.

	Polynomial Equation	*Value of x*
49.	$x^3 - 7x + 6 = 0$	$x = 2$
50.	$x^3 - 28x - 48 = 0$	$x = -4$
51.	$2x^3 - 15x^2 + 27x - 10 = 0$	$x = \frac{1}{2}$
52.	$48x^3 - 80x^2 + 41x - 6 = 0$	$x = \frac{2}{3}$
53.	$x^3 + 2x^2 - 3x - 6 = 0$	$x = \sqrt{3}$
54.	$x^3 + 2x^2 - 2x - 4 = 0$	$x = \sqrt{2}$
55.	$x^3 - 3x^2 + 2 = 0$	$x = 1 + \sqrt{3}$
56.	$x^3 - x^2 - 13x - 3 = 0$	$x = 2 - \sqrt{5}$

In Exercises 57–64, (a) verify the given factors of the function *f*, (b) find the remaining factors of *f*, (c) use your results to write the complete factorization of *f*, (d) list all real zeros of *f*, and (e) confirm your results by using a graphing utility to graph the function.

	Function	*Factors*
57.	$f(x) = 2x^3 + x^2 - 5x + 2$	$(x + 2), (x - 1)$
58.	$f(x) = 3x^3 + 2x^2 - 19x + 6$	$(x + 3), (x - 2)$
59.	$f(x) = x^4 - 4x^3 - 15x^2 + 58x - 40$	$(x - 5), (x + 4)$
60.	$f(x) = 8x^4 - 14x^3 - 71x^2 - 10x + 24$	$(x + 2), (x - 4)$
61.	$f(x) = 6x^3 + 41x^2 - 9x - 14$	$(2x + 1), (3x - 2)$
62.	$f(x) = 10x^3 - 11x^2 - 72x + 45$	$(2x + 5), (5x - 3)$
63.	$f(x) = 2x^3 - x^2 - 10x + 5$	$(2x - 1), (x + \sqrt{5})$
64.	$f(x) = x^3 + 3x^2 - 48x - 144$	$(x + 4\sqrt{3}), (x + 3)$

Graphical Analysis **In Exercises 65–68, (a) use the *zero* or *root* feature of a graphing utility to approximate the zeros of the function accurate to three decimal places, (b) determine one of the exact zeros, and (c) use synthetic division to verify your result from part (b), and then factor the polynomial completely.**

65. $f(x) = x^3 - 2x^2 - 5x + 10$
66. $g(x) = x^3 - 4x^2 - 2x + 8$
67. $h(t) = t^3 - 2t^2 - 7t + 2$
68. $f(s) = s^3 - 12s^2 + 40s - 24$

In Exercises 69–72, simplify the rational expression by using long division or synthetic division.

69. $\dfrac{4x^3 - 8x^2 + x + 3}{2x - 3}$

70. $\dfrac{x^3 + x^2 - 64x - 64}{x + 8}$

71. $\dfrac{x^4 + 6x^3 + 11x^2 + 6x}{x^2 + 3x + 2}$

72. $\dfrac{x^4 + 9x^3 - 5x^2 - 36x + 4}{x^2 - 4}$

Model It

73. ***Data Analysis: Military Personnel*** The numbers M (in thousands) of United States military personnel on active duty for the years 1993 through 2003 are shown in the table, where t represents the year, with $t = 3$ corresponding to 1993. (Source: U.S. Department of Defense)

Year, t	Military personnel, M
3	1705
4	1611
5	1518
6	1472
7	1439
8	1407
9	1386
10	1384
11	1385
12	1412
13	1434

2.4 Exercises

VOCABULARY CHECK:

1. Match the type of complex number with its definition.

(a) Real Number (i) $a + bi, \quad a \neq 0, \quad b \neq 0$

(b) Imaginary number (ii) $a + bi, \quad a = 0, \quad b \neq 0$

(c) Pure imaginary number (iii) $a + bi, \quad b = 0$

In Exercises 2–5, fill in the blanks.

2. The imaginary unit i is defined as $i =$ ________, where $i^2 =$ ________.

3. If a is a positive number, the ________ ________ root of the negative number $-a$ is defined as $\sqrt{-a} = \sqrt{a}\,i$.

4. The numbers $a + bi$ and $a - bi$ are called ________ ________, and their product is a real number $a^2 + b^2$.

PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at **www.Eduspace.com.**

In Exercises 1–4, find real numbers a and b such that the equation is true.

1. $a + bi = -10 + 6i$
2. $a + bi = 13 + 4i$
3. $(a - 1) + (b + 3)i = 5 + 8i$
4. $(a + 6) + 2bi = 6 - 5i$

In Exercises 5–16, write the complex number in standard form.

5. $4 + \sqrt{-9}$
6. $3 + \sqrt{-16}$
7. $2 - \sqrt{-27}$
8. $1 + \sqrt{-8}$
9. $\sqrt{-75}$
10. $\sqrt{-4}$
11. 8
12. 45
13. $-6i + i^2$
14. $-4i^2 + 2i$
15. $\sqrt{-0.09}$
16. $\sqrt{-0.0004}$

In Exercises 17–26, perform the addition or subtraction and write the result in standard form.

17. $(5 + i) + (6 - 2i)$
18. $(13 - 2i) + (-5 + 6i)$
19. $(8 - i) - (4 - i)$
20. $(3 + 2i) - (6 + 13i)$
21. $\left(-2 + \sqrt{-8}\right) + \left(5 - \sqrt{-50}\right)$
22. $\left(8 + \sqrt{-18}\right) - \left(4 + 3\sqrt{2}i\right)$
23. $13i - (14 - 7i)$
24. $22 + (-5 + 8i) + 10i$
25. $-\left(\frac{3}{2} + \frac{5}{2}i\right) + \left(\frac{5}{3} + \frac{11}{3}i\right)$
26. $(1.6 + 3.2i) + (-5.8 + 4.3i)$

In Exercises 27–36, perform the operation and write the result in standard form.

27. $(1 + i)(3 - 2i)$
28. $(6 - 2i)(2 - 3i)$
29. $6i(5 - 2i)$
30. $-8i(9 + 4i)$
31. $\left(\sqrt{14} + \sqrt{10}i\right)\left(\sqrt{14} - \sqrt{10}i\right)$
32. $\left(\sqrt{3} + \sqrt{15}i\right)\left(\sqrt{3} - \sqrt{15}i\right)$
33. $(4 + 5i)^2$
34. $(2 - 3i)^2$
35. $(2 + 3i)^2 + (2 - 3i)^2$
36. $(1 - 2i)^2 - (1 + 2i)^2$

In Exercises 37–44, write the complex conjugate of the complex number. Then multiply the number by its complex conjugate.

37. $6 + 3i$
38. $7 - 12i$
39. $-1 - \sqrt{5}i$
40. $-3 + \sqrt{2}i$
41. $\sqrt{-20}$
42. $\sqrt{-15}$
43. $\sqrt{8}$
44. $1 + \sqrt{8}$

In Exercises 45–54, write the quotient in standard form.

45. $\dfrac{5}{i}$
46. $-\dfrac{14}{2i}$
47. $\dfrac{2}{4 - 5i}$
48. $\dfrac{5}{1 - i}$
49. $\dfrac{3 + i}{3 - i}$
50. $\dfrac{6 - 7i}{1 - 2i}$
51. $\dfrac{6 - 5i}{i}$
52. $\dfrac{8 + 16i}{2i}$
53. $\dfrac{3i}{(4 - 5i)^2}$
54. $\dfrac{5i}{(2 + 3i)^2}$

In Exercises 55–58, perform the operation and write the result in standard form.

55. $\dfrac{2}{1 + i} - \dfrac{3}{1 - i}$
56. $\dfrac{2i}{2 + i} + \dfrac{5}{2 - i}$
57. $\dfrac{i}{3 - 2i} + \dfrac{2i}{3 + 8i}$
58. $\dfrac{1 + i}{i} - \dfrac{3}{4 - i}$

In Exercises 59–64, write the complex number in standard form.

59. $\sqrt{-6}\cdot\sqrt{-2}$
60. $\sqrt{-5}\cdot\sqrt{-10}$
61. $\left(\sqrt{-10}\right)^2$
62. $\left(\sqrt{-75}\right)^2$
63. $\left(3+\sqrt{-5}\right)\left(7-\sqrt{-10}\right)$
64. $\left(2-\sqrt{-6}\right)^2$

In Exercises 65–74, use the Quadratic Formula to solve the quadratic equation.

65. $x^2 - 2x + 2 = 0$
66. $x^2 + 6x + 10 = 0$
67. $4x^2 + 16x + 17 = 0$
68. $9x^2 - 6x + 37 = 0$
69. $4x^2 + 16x + 15 = 0$
70. $16t^2 - 4t + 3 = 0$
71. $\frac{3}{2}x^2 - 6x + 9 = 0$
72. $\frac{7}{8}x^2 - \frac{3}{4}x + \frac{5}{16} = 0$
73. $1.4x^2 - 2x - 10 = 0$
74. $4.5x^2 - 3x + 12 = 0$

In Exercises 75–82, simplify the complex number and write it in standard form.

75. $-6i^3 + i^2$
76. $4i^2 - 2i^3$
77. $-5i^5$
78. $(-i)^3$
79. $\left(\sqrt{-75}\right)^3$
80. $\left(\sqrt{-2}\right)^6$
81. $\dfrac{1}{i^3}$
82. $\dfrac{1}{(2i)^3}$

Model It

83. ***Impedance*** The opposition to current in an electrical circuit is called its impedance. The impedance z in a parallel circuit with two pathways satisfies the equation

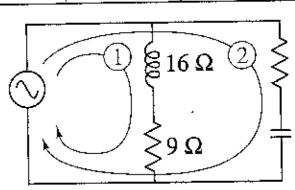
$$\frac{1}{z} = \frac{1}{z_1} + \frac{1}{z_2}$$

where z_1 is the impedance (in ohms) of pathway 1 and z_2 is the impedance of pathway 2.

(a) The impedance of each pathway in a parallel circuit is found by adding the impedances of all components in the pathway. Use the table to find z_1 and z_2.

(b) Find the impedance z.

	Resistor	Inductor	Capacitor
Symbol	$a\Omega$	$b\Omega$	$c\Omega$
Impedance	a	bi	$-ci$



84. Cube each complex number.
(a) 2 (b) $-1 + \sqrt{3}i$ (c) $-1 - \sqrt{3}i$

85. Raise each complex number to the fourth power.
(a) 2 (b) -2 (c) $2i$ (d) $-2i$

86. Write each of the powers of i as i, $-i$, 1, or -1.
(a) i^{40} (b) i^{25} (c) i^{50} (d) i^{67}

Synthesis

True or False? **In Exercises 87–89, determine whether the statement is true or false. Justify your answer.**

87. There is no complex number that is equal to its complex conjugate.

88. $-i\sqrt{6}$ is a solution of $x^4 - x^2 + 14 = 56$.

89. $i^{44} + i^{150} - i^{74} - i^{109} + i^{61} = -1$

90. ***Error Analysis*** Describe the error.

~~$\sqrt{-6}\sqrt{-6} = \sqrt{(-6)(-6)} = \sqrt{36} = 6$~~

91. ***Proof*** Prove that the complex conjugate of the product of two complex numbers $a_1 + b_1i$ and $a_2 + b_2i$ is the product of their complex conjugates.

92. ***Proof*** Prove that the complex conjugate of the sum of two complex numbers $a_1 + b_1i$ and $a_2 + b_2i$ is the sum of their complex conjugates.

Skills Review

In Exercises 93–96, perform the operation and write the result in standard form.

93. $(4 + 3x) + (8 - 6x - x^2)$
94. $(x^3 - 3x^2) - (6 - 2x - 4x^2)$
95. $\left(3x - \frac{1}{2}\right)(x + 4)$
96. $(2x - 5)^2$

In Exercises 97–100, solve the equation and check your solution.

97. $-x - 12 = 19$
98. $8 - 3x = -34$
99. $4(5x - 6) - 3(6x + 1) = 0$
100. $5[x - (3x + 11)] = 20x - 15$

101. ***Volume of an Oblate Spheroid***
Solve for a: $V = \frac{4}{3}\pi a^2 b$

102. ***Newton's Law of Universal Gravitation***
Solve for r: $F = \alpha\dfrac{m_1 m_2}{r^2}$

103. ***Mixture Problem*** A five-liter container contains a mixture with a concentration of 50%. How much of this mixture must be withdrawn and replaced by 100% concentrate to bring the mixture up to 60% concentration?

2.5 Exercises

VOCABULARY CHECK: Fill in the blanks.

1. The ________ ________ of ________ states that if $f(x)$ is a polynomial of degree n $(n > 0)$, then f has at least one zero in the complex number system.
2. The ________ ________ ________ states that if $f(x)$ is a polynomial of degree n $(n > 0)$, then f has precisely n linear factors $f(x) = a_n(x - c_1)(x - c_2) \cdot \cdot \cdot (x - c_n)$ where $c_1, c_2, \ldots, c_n$ are complex numbers.
3. The test that gives a list of the possible rational zeros of a polynomial function is called the ________ ________ Test.
4. If $a + bi$ is a complex zero of a polynomial with real coefficients, then so is its ________, $a - bi$.
5. A quadratic factor that cannot be factored further as a product of linear factors containing real numbers is said to be ________ over the ________.
6. The theorem that can be used to determine the possible numbers of positive real zeros and negative real zeros of a function is called ________ ________ of ________.
7. A real number b is a(n) ________ bound for the real zeros of f if no real zeros are less than b, and is a(n) ________ bound if no real zeros are greater than b.

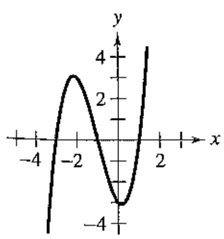
PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at **www.Eduspace.com.**

In Exercises 1–6, find all the zeros of the function.

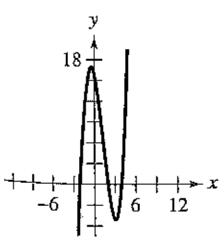
1. $f(x) = x(x - 6)^2$
2. $f(x) = x^2(x + 3)(x^2 - 1)$
3. $g(x) = (x - 2)(x + 4)^3$
4. $f(x) = (x + 5)(x - 8)^2$
5. $f(x) = (x + 6)(x + i)(x - i)$
6. $h(t) = (t - 3)(t - 2)(t - 3i)(t + 3i)$

In Exercises 7–10, use the Rational Zero Test to list all possible rational zeros of f. Verify that the zeros of f shown on the graph are contained in the list.

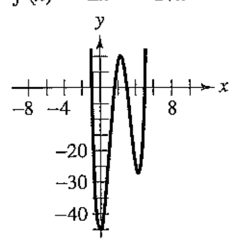
7. $f(x) = x^3 + 3x^2 - x - 3$



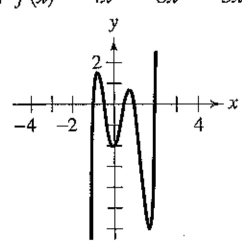
8. $f(x) = x^3 - 4x^2 - 4x + 16$



9. $f(x) = 2x^4 - 17x^3 + 35x^2 + 9x - 45$



10. $f(x) = 4x^5 - 8x^4 - 5x^3 + 10x^2 + x - 2$



In Exercises 11–20, find all the rational zeros of the function.

11. $f(x) = x^3 - 6x^2 + 11x - 6$
12. $f(x) = x^3 - 7x - 6$
13. $g(x) = x^3 - 4x^2 - x + 4$
14. $h(x) = x^3 - 9x^2 + 20x - 12$
15. $h(t) = t^3 + 12t^2 + 21t + 10$
16. $p(x) = x^3 - 9x^2 + 27x - 27$
17. $C(x) = 2x^3 + 3x^2 - 1$
18. $f(x) = 3x^3 - 19x^2 + 33x - 9$
19. $f(x) = 9x^4 - 9x^3 - 58x^2 + 4x + 24$
20. $f(x) = 2x^4 - 15x^3 + 23x^2 + 15x - 25$

In Exercises 21–24, find all real solutions of the polynomial equation.

21. $z^4 - z^3 - 2z - 4 = 0$

22. $x^4 - 13x^2 - 12x = 0$

23. $2y^4 + 7y^3 - 26y^2 + 23y - 6 = 0$

24. $x^5 - x^4 - 3x^3 + 5x^2 - 2x = 0$

In Exercises 25–28, (a) list the possible rational zeros of *f*, (b) sketch the graph of *f* so that some of the possible zeros in part (a) can be disregarded, and then (c) determine all real zeros of *f*.

25. $f(x) = x^3 + x^2 - 4x - 4$

26. $f(x) = -3x^3 + 20x^2 - 36x + 16$

27. $f(x) = -4x^3 + 15x^2 - 8x - 3$

28. $f(x) = 4x^3 - 12x^2 - x + 15$

In Exercises 29–32, (a) list the possible rational zeros of *f*, (b) use a graphing utility to graph *f* so that some of the possible zeros in part (a) can be disregarded, and then (c) determine all real zeros of *f*.

29. $f(x) = -2x^4 + 13x^3 - 21x^2 + 2x + 8$

30. $f(x) = 4x^4 - 17x^2 + 4$

31. $f(x) = 32x^3 - 52x^2 + 17x + 3$

32. $f(x) = 4x^3 + 7x^2 - 11x - 18$

Graphical Analysis **In Exercises 33–36, (a) use the *zero* or *root* feature of a graphing utility to approximate the zeros of the function accurate to three decimal places, (b) determine one of the exact zeros (use synthetic division to verify your result), and (c) factor the polynomial completely.**

33. $f(x) = x^4 - 3x^2 + 2$

34. $P(t) = t^4 - 7t^2 + 12$

35. $h(x) = x^5 - 7x^4 + 10x^3 + 14x^2 - 24x$

36. $g(x) = 6x^4 - 11x^3 - 51x^2 + 99x - 27$

In Exercises 37–42, find a polynomial function with real coefficients that has the given zeros. (There are many correct answers.)

37. $1, 5i, -5i$

38. $4, 3i, -3i$

39. $6, -5 + 2i, -5 - 2i$

40. $2, 4 + i, 4 - i$

41. $\frac{2}{3}, -1, 3 + \sqrt{2}i$

42. $-5, -5, 1 + \sqrt{3}i$

In Exercises 43–46, write the polynomial (a) as the product of factors that are irreducible over the *rationals*, (b) as the product of linear and quadratic factors that are irreducible over the *reals*, and (c) in completely factored form.

43. $f(x) = x^4 + 6x^2 - 27$

44. $f(x) = x^4 - 2x^3 - 3x^2 + 12x - 18$
(*Hint:* One factor is $x^2 - 6$.)

45. $f(x) = x^4 - 4x^3 + 5x^2 - 2x - 6$
(*Hint:* One factor is $x^2 - 2x - 2$.)

46. $f(x) = x^4 - 3x^3 - x^2 - 12x - 20$
(*Hint:* One factor is $x^2 + 4$.)

In Exercises 47–54, use the given zero to find all the zeros of the function.

Function	*Zero*
47. $f(x) = 2x^3 + 3x^2 + 50x + 75$	$5i$
48. $f(x) = x^3 + x^2 + 9x + 9$	$3i$
49. $f(x) = 2x^4 - x^3 + 7x^2 - 4x - 4$	$2i$
50. $g(x) = x^3 - 7x^2 - x + 87$	$5 + 2i$
51. $g(x) = 4x^3 + 23x^2 + 34x - 10$	$-3 + i$
52. $h(x) = 3x^3 - 4x^2 + 8x + 8$	$1 - \sqrt{3}i$
53. $f(x) = x^4 + 3x^3 - 5x^2 - 21x + 22$	$-3 + \sqrt{2}i$
54. $f(x) = x^3 + 4x^2 + 14x + 20$	$-1 - 3i$

In Exercises 55–72, find all the zeros of the function and write the polynomial as a product of linear factors.

55. $f(x) = x^2 + 25$

56. $f(x) = x^2 - x + 56$

57. $h(x) = x^2 - 4x + 1$

58. $g(x) = x^2 + 10x + 23$

59. $f(x) = x^4 - 81$

60. $f(y) = y^4 - 625$

61. $f(z) = z^2 - 2z + 2$

62. $h(x) = x^3 - 3x^2 + 4x - 2$

63. $g(x) = x^3 - 6x^2 + 13x - 10$

64. $f(x) = x^3 - 2x^2 - 11x + 52$

65. $h(x) = x^3 - x + 6$

66. $h(x) = x^3 + 9x^2 + 27x + 35$

67. $f(x) = 5x^3 - 9x^2 + 28x + 6$

68. $g(x) = 3x^3 - 4x^2 + 8x + 8$

69. $g(x) = x^4 - 4x^3 + 8x^2 - 16x + 16$

70. $h(x) = x^4 + 6x^3 + 10x^2 + 6x + 9$

71. $f(x) = x^4 + 10x^2 + 9$

72. $f(x) = x^4 + 29x^2 + 100$

In Exercises 73–78, find all the zeros of the function. When there is an extended list of possible rational zeros, use a graphing utility to graph the function in order to discard any rational zeros that are obviously not zeros of the function.

73. $f(x) = x^3 + 24x^2 + 214x + 740$

74. $f(s) = 2s^3 - 5s^2 + 12s - 5$

75. $f(x) = 16x^3 - 20x^2 - 4x + 15$

76. $f(x) = 9x^3 - 15x^2 + 11x - 5$

77. $f(x) = 2x^4 + 5x^3 + 4x^2 + 5x + 2$

78. $g(x) = x^5 - 8x^4 + 28x^3 - 56x^2 + 64x - 32$

In Exercises 79–86, use Descartes's Rule of Signs to determine the possible numbers of positive and negative zeros of the function.

79. $g(x) = 5x^5 + 10x$
80. $h(x) = 4x^2 - 8x + 3$
81. $h(x) = 3x^4 + 2x^2 + 1$
82. $h(x) = 2x^4 - 3x + 2$
83. $g(x) = 2x^3 - 3x^2 - 3$
84. $f(x) = 4x^3 - 3x^2 + 2x - 1$
85. $f(x) = -5x^3 + x^2 - x + 5$
86. $f(x) = 3x^3 + 2x^2 + x + 3$

In Exercises 87–90, use synthetic division to verify the upper and lower bounds of the real zeros of f.

87. $f(x) = x^4 - 4x^3 + 15$
(a) Upper: $x = 4$ (b) Lower: $x = -1$

88. $f(x) = 2x^3 - 3x^2 - 12x + 8$
(a) Upper: $x = 4$ (b) Lower: $x = -3$

89. $f(x) = x^4 - 4x^3 + 16x - 16$
(a) Upper: $x = 5$ (b) Lower: $x = -3$

90. $f(x) = 2x^4 - 8x + 3$
(a) Upper: $x = 3$ (b) Lower: $x = -4$

In Exercises 91–94, find all the real zeros of the function.

91. $f(x) = 4x^3 - 3x - 1$
92. $f(z) = 12z^3 - 4z^2 - 27z + 9$
93. $f(y) = 4y^3 + 3y^2 + 8y + 6$
94. $g(x) = 3x^3 - 2x^2 + 15x - 10$

In Exercises 95–98, find all the rational zeros of the polynomial function.

95. $P(x) = x^4 - \frac{25}{4}x^2 + 9 = \frac{1}{4}(4x^4 - 25x^2 + 36)$
96. $f(x) = x^3 - \frac{3}{2}x^2 - \frac{23}{2}x + 6 = \frac{1}{2}(2x^3 - 3x^2 - 23x + 12)$
97. $f(x) = x^3 - \frac{1}{4}x^2 - x + \frac{1}{4} = \frac{1}{4}(4x^3 - x^2 - 4x + 1)$
98. $f(z) = z^3 + \frac{11}{6}z^2 - \frac{1}{2}z - \frac{1}{3} = \frac{1}{6}(6z^3 + 11z^2 - 3z - 2)$

In Exercises 99–102, match the cubic function with the numbers of rational and irrational zeros.

(a) Rational zeros: 0; irrational zeros: 1
(b) Rational zeros: 3; irrational zeros: 0
(c) Rational zeros: 1; irrational zeros: 2
(d) Rational zeros: 1; irrational zeros: 0

99. $f(x) = x^3 - 1$
100. $f(x) = x^3 - 2$
101. $f(x) = x^3 - x$
102. $f(x) = x^3 - 2x$

103. ***Geometry*** An open box is to be made from a rectangular piece of material, 15 centimeters by 9 centimeters, by cutting equal squares from the corners and turning up the sides.

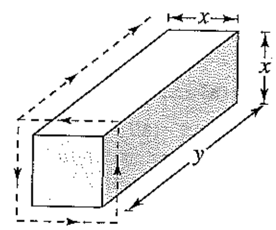
(a) Let x represent the length of the sides of the squares removed. Draw a diagram showing the squares removed from the original piece of material and the resulting dimensions of the open box.

(b) Use the diagram to write the volume V of the box as a function of x. Determine the domain of the function.

(c) Sketch the graph of the function and approximate the dimensions of the box that will yield a maximum volume.

(d) Find values of x such that $V = 56$. Which of these values is a physical impossibility in the construction of the box? Explain.

104. ***Geometry*** A rectangular package to be sent by a delivery service (see figure) can have a maximum combined length and girth (perimeter of a cross section) of 120 inches.



(a) Show that the volume of the package is

$$V(x) = 4x^2(30 - x).$$

(b) Use a graphing utility to graph the function and approximate the dimensions of the package that will yield a maximum volume.

(c) Find values of x such that $V = 13{,}500$. Which of these values is a physical impossibility in the construction of the package? Explain.

105. ***Advertising Cost*** A company that produces MP3 players estimates that the profit P (in dollars) for selling a particular model is given by

$$P = -76x^3 + 4830x^2 - 320{,}000, \quad 0 \le x \le 60$$

where x is the advertising expense (in tens of thousands of dollars). Using this model, find the smaller of two advertising amounts that will yield a profit of \$2,500,000.

106. ***Advertising Cost*** A company that manufactures bicycles estimates that the profit P (in dollars) for selling a particular model is given by

$$P = -45x^3 + 2500x^2 - 275{,}000, \quad 0 \le x \le 50$$

where x is the advertising expense (in tens of thousands of dollars). Using this model, find the smaller of two advertising amounts that will yield a profit of \$800,000.

107. ***Geometry*** A bulk food storage bin with dimensions 2 feet by 3 feet by 4 feet needs to be increased in size to hold five times as much food as the current bin. (Assume each dimension is increased by the same amount.)

(a) Write a function that represents the volume V of the new bin.

(b) Find the dimensions of the new bin.

108. ***Geometry*** A rancher wants to enlarge an existing rectangular corral such that the total area of the new corral is 1.5 times that of the original corral. The current corral's dimensions are 250 feet by 160 feet. The rancher wants to increase each dimension by the same amount.

(a) Write a function that represents the area A of the new corral.

(b) Find the dimensions of the new corral.

(c) A rancher wants to add a length to the sides of the corral that are 160 feet, and twice the length to the sides that are 250 feet, such that the total area of the new corral is 1.5 times that of the original corral. Repeat parts (a) and (b). Explain your results.

109. ***Cost*** The ordering and transportation cost C (in thousands of dollars) for the components used in manufacturing a product is given by

$$C = 100\left(\frac{200}{x^2} + \frac{x}{x+30}\right), \quad x \geq 1$$

where x is the order size (in hundreds). In calculus, it can be shown that the cost is a minimum when

$$3x^3 - 40x^2 - 2400x - 36{,}000 = 0.$$

Use a calculator to approximate the optimal order size to the nearest hundred units.

110. ***Height of a Baseball*** A baseball is thrown upward from a height of 6 feet with an initial velocity of 48 feet per second, and its height h (in feet) is

$$h(t) = -16t^2 + 48t + 6, \quad 0 \leq t \leq 3$$

where t is the time (in seconds). You are told the ball reaches a height of 64 feet. Is this possible?

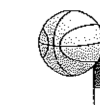
111. ***Profit*** The demand equation for a certain product is $p = 140 - 0.0001x$, where p is the unit price (in dollars) of the product and x is the number of units produced and sold. The cost equation for the product is $C = 80x + 150{,}000$, where C is the total cost (in dollars) and x is the number of units produced. The total profit obtained by producing and selling x units is

$$P = R - C = xp - C.$$

You are working in the marketing department of the company that produces this product, and you are asked to determine a price p that will yield a profit of 9 million dollars. Is this possible? Explain.

Model It

112. ***Athletics*** The attendance A (in millions) at NCAA women's college basketball games for the years 1997 through 2003 is shown in the table, where t represents the year, with $t = 7$ corresponding to 1997. (Source: National Collegiate Athletic Association)



Year, t	Attendance, A
7	6.7
8	7.4
9	8.0
10	8.7
11	8.8
12	9.5
13	10.2

(a) Use the *regression* feature of a graphing utility to find a cubic model for the data.

(b) Use the graphing utility to create a scatter plot of the data. Then graph the model and the scatter plot in the same viewing window. How do they compare?

(c) According to the model found in part (a), in what year did attendance reach 8.5 million?

(d) According to the model found in part (a), in what year did attendance reach 9 million?

(e) According to the right-hand behavior of the model, will the attendance continue to increase? Explain.

Synthesis

True or False? **In Exercises 113 and 114, decide whether the statement is true or false. Justify your answer.**

113. It is possible for a third-degree polynomial function with integer coefficients to have no real zeros.

114. If $x = -i$ is a zero of the function given by

$$f(x) = x^3 + ix^2 + ix - 1$$

then $x = i$ must also be a zero of f.

Think About It **In Exercises 115–120, determine (if possible) the zeros of the function g if the function f has zeros at $x = r_1, x = r_2$, and $x = r_3$.**

115. $g(x) = -f(x)$

116. $g(x) = 3f(x)$

2.6 Exercises

VOCABULARY CHECK: Fill in the blanks.

1. Functions of the form $f(x) = N(x)/D(x)$, where $N(x)$ and $D(x)$ are polynomials and $D(x)$ is not the zero polynomial, are called ________ ________.
2. If $f(x) \to \pm\infty$ as $x \to a$ from the left or the right, then $x = a$ is a ________ ________ of the graph of f.
3. If $f(x) \to b$ as $x \to \pm\infty$, then $y = b$ is a ________ ________ of the graph of f.
4. For the rational function given by $f(x) = N(x)/D(x)$, if the degree of $N(x)$ is exactly one more than the degree of $D(x)$, then the graph of f has a ________ (or oblique) ________.

PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at **www.Eduspace.com.**

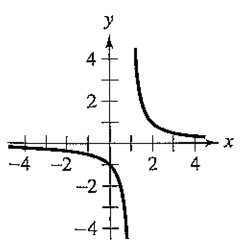
In Exercises 1–4, (a) complete each table for the function, (b) determine the vertical and horizontal asymptotes of the graph of the function, and (c) find the domain of the function.

x	$f(x)$
0.5	
0.9	
0.99	
0.999	

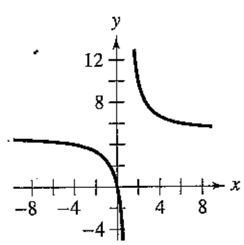
x	$f(x)$
1.5	
1.1	
1.01	
1.001	

x	$f(x)$
5	
10	
100	
1000	

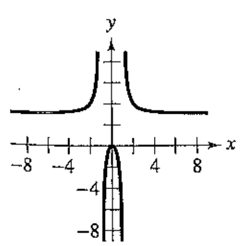
1. $f(x) = \dfrac{1}{x-1}$



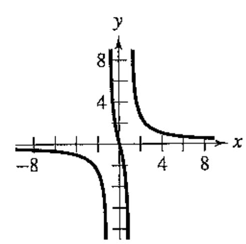
2. $f(x) = \dfrac{5x}{x-1}$



3. $f(x) = \dfrac{3x^2}{x^2-1}$



4. $f(x) = \dfrac{4x}{x^2-1}$

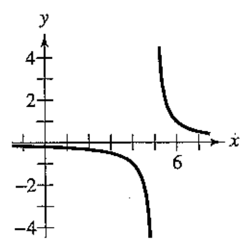


In Exercises 5–12, find the domain of the function and identify any horizontal and vertical asymptotes.

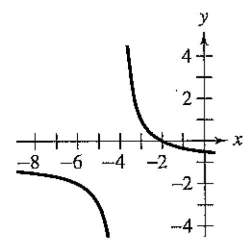
5. $f(x) = \dfrac{1}{x^2}$
6. $f(x) = \dfrac{4}{(x-2)^3}$
7. $f(x) = \dfrac{2+x}{2-x}$
8. $f(x) = \dfrac{1-5x}{1+2x}$
9. $f(x) = \dfrac{x^3}{x^2-1}$
10. $f(x) = \dfrac{2x^2}{x+1}$
11. $f(x) = \dfrac{3x^2+1}{x^2+x+9}$
12. $f(x) = \dfrac{3x^2+x-5}{x^2+1}$

In Exercises 13–16, match the rational function with its graph. [The graphs are labeled (a), (b), (c), and (d).]

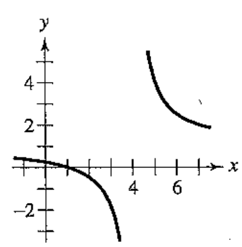
(a)



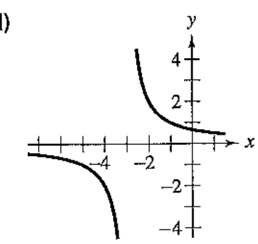
(b)



(c)



(d)



13. $f(x) = \dfrac{2}{x+3}$
14. $f(x) = \dfrac{1}{x-5}$
15. $f(x) = \dfrac{x-1}{x-4}$
16. $f(x) = -\dfrac{x+2}{x+4}$

In Exercises 17–20, find the zeros (if any) of the rational function.

17. $g(x) = \dfrac{x^2-1}{x+1}$
18. $h(x) = 2 + \dfrac{5}{x^2+2}$
19. $f(x) = 1 - \dfrac{3}{x-3}$
20. $g(x) = \dfrac{x^3-8}{x^2+1}$

In Exercises 21–26, find the domain of the function and identify any horizontal and vertical asymptotes.

21. $f(x) = \dfrac{x-4}{x^2-16}$

22. $f(x) = \dfrac{x+3}{x^2-9}$

23. $f(x) = \dfrac{x^2-1}{x^2-2x-3}$

24. $f(x) = \dfrac{x^2-4}{x^2-3x+2}$

25. $f(x) = \dfrac{x^2-3x-4}{2x^2+x-1}$

26. $f(x) = \dfrac{6x^2-11x+3}{6x^2-7x-3}$

In Exercises 27–46, (a) state the domain of the function, (b) identify all intercepts, (c) find any vertical and horizontal asymptotes, and (d) plot additional solution points as needed to sketch the graph of the rational function.

27. $f(x) = \dfrac{1}{x+2}$

28. $f(x) = \dfrac{1}{x-3}$

29. $h(x) = \dfrac{-1}{x+2}$

30. $g(x) = \dfrac{1}{3-x}$

31. $C(x) = \dfrac{5+2x}{1+x}$

32. $P(x) = \dfrac{1-3x}{1-x}$

33. $f(x) = \dfrac{x^2}{x^2+9}$

34. $f(t) = \dfrac{1-2t}{t}$

35. $g(s) = \dfrac{s}{s^2+1}$

36. $f(x) = -\dfrac{1}{(x-2)^2}$

37. $h(x) = \dfrac{x^2-5x+4}{x^2-4}$

38. $g(x) = \dfrac{x^2-2x-8}{x^2-9}$

39. $f(x) = \dfrac{2x^2-5x-3}{x^3-2x^2-x+2}$

40. $f(x) = \dfrac{x^2-x-2}{x^3-2x^2-5x+6}$

41. $f(x) = \dfrac{x^2+3x}{x^2+x-6}$

42. $f(x) = \dfrac{5(x+4)}{x^2+x-12}$

43. $f(x) = \dfrac{2x^2-5x+2}{2x^2-x-6}$

44. $f(x) = \dfrac{3x^2-8x+4}{2x^2-3x-2}$

45. $f(t) = \dfrac{t^2-1}{t+1}$

46. $f(x) = \dfrac{x^2-16}{x-4}$

Analytical, Numerical, and Graphical Analysis **In Exercises 47–50, do the following.**

(a) Determine the domains of *f* and *g*.

(b) Simplify *f* and find any vertical asymptotes of the graph of *f*.

(c) Compare the functions by completing the table.

(d) Use a graphing utility to graph *f* and *g* in the same viewing window.

(e) Explain why the graphing utility may not show the difference in the domains of *f* and *g*.

47. $f(x) = \dfrac{x^2-1}{x+1}$, $g(x) = x-1$

x	-3	-2	-1.5	-1	-0.5	0	1
$f(x)$							
$g(x)$							

48. $f(x) = \dfrac{x^2(x-2)}{x^2-2x}$, $g(x) = x$

x	-1	0	1	1.5	2	2.5	3
$f(x)$							
$g(x)$							

49. $f(x) = \dfrac{x-2}{x^2-2x}$, $g(x) = \dfrac{1}{x}$

x	-0.5	0	0.5	1	1.5	2	3
$f(x)$							
$g(x)$							

50. $f(x) = \dfrac{2x-6}{x^2-7x+12}$, $g(x) = \dfrac{2}{x-4}$

x	0	1	2	3	4	5	6
$f(x)$							
$g(x)$							

In Exercises 51–64, (a) state the domain of the function, (b) identify all intercepts, (c) identify any vertical and slant asymptotes, and (d) plot additional solution points as needed to sketch the graph of the rational function.

51. $h(x) = \dfrac{x^2-4}{x}$

52. $g(x) = \dfrac{x^2+5}{x}$

53. $f(x) = \dfrac{2x^2+1}{x}$

54. $f(x) = \dfrac{1-x^2}{x}$

55. $g(x) = \dfrac{x^2+1}{x}$

56. $h(x) = \dfrac{x^2}{x-1}$

57. $f(t) = -\dfrac{t^2+1}{t+5}$

58. $f(x) = \dfrac{x^2}{3x+1}$

59. $f(x) = \dfrac{x^3}{x^2-1}$

60. $g(x) = \dfrac{x^3}{2x^2-8}$

61. $f(x) = \dfrac{x^2-x+1}{x-1}$

62. $f(x) = \dfrac{2x^2-5x+5}{x-2}$

63. $f(x) = \dfrac{2x^3 - x^2 - 2x + 1}{x^2 + 3x + 2}$

64. $f(x) = \dfrac{2x^3 + x^2 - 8x - 4}{x^2 - 3x + 2}$

In Exercises 65–68, use a graphing utility to graph the rational function. Give the domain of the function and identify any asymptotes. Then zoom out sufficiently far so that the graph appears as a line. Identify the line.

65. $f(x) = \dfrac{x^2 + 5x + 8}{x + 3}$

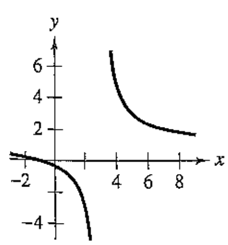
66. $f(x) = \dfrac{2x^2 + x}{x + 1}$

67. $g(x) = \dfrac{1 + 3x^2 - x^3}{x^2}$

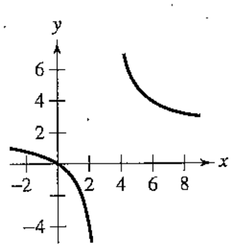
68. $h(x) = \dfrac{12 - 2x - x^2}{2(4 + x)}$

Graphical Reasoning **In Exercises 69–72, (a) use the graph to determine any x-intercepts of the graph of the rational function and (b) set $y = 0$ and solve the resulting equation to confirm your result in part (a).**

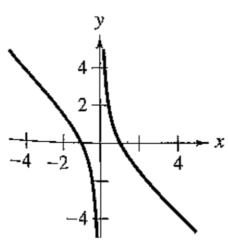
69. $y = \dfrac{x + 1}{x - 3}$



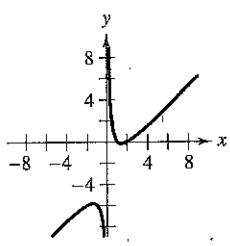
70. $y = \dfrac{2x}{x - 3}$



71. $y = \dfrac{1}{x} - x$



72. $y = x - 3 + \dfrac{2}{x}$



73. ***Pollution*** The cost C (in millions of dollars) of removing $p\%$ of the industrial and municipal pollutants discharged into a river is given by

$$C = \frac{255p}{100 - p}, \quad 0 \le p < 100.$$

(a) Use a graphing utility to graph the cost function.

(b) Find the costs of removing 10%, 40%, and 75% of the pollutants.

(c) According to this model, would it be possible to remove 100% of the pollutants? Explain.

74. ***Recycling*** In a pilot project, a rural township is given recycling bins for separating and storing recyclable products. The cost C (in dollars) for supplying bins to $p\%$ of the population is given by

$$C = \frac{25{,}000p}{100 - p}, \quad 0 \le p < 100.$$

(a) Use a graphing utility to graph the cost function.

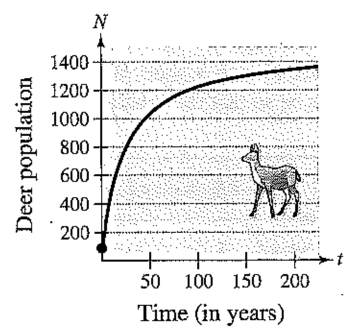
(b) Find the costs of supplying bins to 15%, 50%, and 90% of the population.

(c) According to this model, would it be possible to supply bins to 100% of the residents? Explain.

75. ***Population Growth*** The game commission introduces 100 deer into newly acquired state game lands. The population N of the herd is modeled by

$$N = \frac{20(5 + 3t)}{1 + 0.04t}, \quad t \ge 0$$

where t is the time in years (see figure).



(a) Find the populations when $t = 5$, $t = 10$, and $t = 25$.

(b) What is the limiting size of the herd as time increases?

76. ***Concentration of a Mixture*** A 1000-liter tank contains 50 liters of a 25% brine solution. You add x liters of a 75% brine solution to the tank.

(a) Show that the concentration C, the proportion of brine to total solution, in the final mixture is

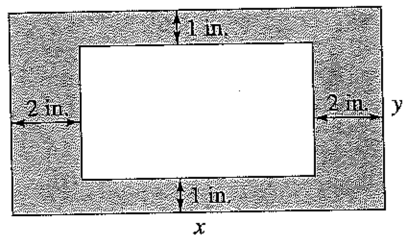
$$C = \frac{3x + 50}{4(x + 50)}.$$

(b) Determine the domain of the function based on the physical constraints of the problem.

(c) Sketch a graph of the concentration function.

(d) As the tank is filled, what happens to the rate at which the concentration of brine is increasing? What percent does the concentration of brine appear to approach?

77. ***Page Design*** A page that is x inches wide and y inches high contains 30 square inches of print. The top and bottom margins are 1 inch deep and the margins on each side are 2 inches wide (see figure).



(a) Show that the total area A on the page is

$$A = \frac{2x(x + 11)}{x - 4}.$$

(b) Determine the domain of the function based on the physical constraints of the problem.

(c) Use a graphing utility to graph the area function and approximate the page size for which the least amount of paper will be used. Verify your answer numerically using the *table* feature of the graphing utility.

78. ***Page Design*** A rectangular page is designed to contain 64 square inches of print. The margins at the top and bottom of the page are each 1 inch deep. The margins on each side are $1\frac{1}{2}$ inches wide. What should the dimensions of the page be so that the least amount of paper is used?

Model It

79. ***Average Speed*** A driver averaged 50 miles per hour on the round trip between Akron, Ohio, and Columbus, Ohio, 100 miles away. The average speeds for going and returning were x and y miles per hour, respectively.

(a) Show that $y = \dfrac{25x}{x - 25}$.

(b) Determine the vertical and horizontal asymptotes of the graph of the function.

(c) Use a graphing utility to graph the function.

(d) Complete the table.

x	30	35	40	45	50	55	60
y							

(e) Are the results in the table what you expected? Explain.

(f) Is it possible to average 20 miles per hour in one direction and still average 50 miles per hour on the round trip? Explain.

80. ***Sales*** The sales S (in millions of dollars) for the Yankee Candle Company in the years 1998 through 2003 are shown in the table. (Source: The Yankee Candle Company)

1998	184.5	1999	256.6	2000	338.8
2001	379.8	2002	444.8	2003	508.6

A model for these data is given by

$$S = \frac{5.816t^2 - 130.68}{0.004t^2 + 1.00}, \quad 8 \le t \le 13$$

where t represents the year, with $t = 8$ corresponding to 1998.

(a) Use a graphing utility to plot the data and graph the model in the same viewing window. How well does the model fit the data?

(b) Use the model to estimate the sales for the Yankee Candle Company in 2008.

(c) Would this model be useful for estimating sales after 2008? Explain.

Synthesis

True or False? **In Exercises 81 and 82, determine whether the statement is true or false. Justify your answer.**

81. A polynomial can have infinitely many vertical asymptotes.

82. The graph of a rational function can never cross one of its asymptotes.

Think About It **In Exercises 83 and 84, write a rational function f that has the specified characteristics. (There are many correct answers.)**

83. Vertical asymptote: None
Horizontal asymptote: $y = 2$

84. Vertical asymptote: $x = -2, x = 1$
Horizontal asymptote: None

Skills Review

In Exercises 85–88, completely factor the expression.

85. $x^2 - 15x + 56$

86. $3x^2 + 23x - 36$

87. $x^3 - 5x^2 + 4x - 20$

88. $x^3 + 6x^2 - 2x - 12$

In Exercises 93–96, solve the inequality and graph the solution on the real number line.

89. $10 - 3x \le 0$

90. $5 - 2x > 5(x + 1)$

91. $|4(x - 2)| < 20$

92. $\frac{1}{2}|2x + 3| \ge 5$

93. Make a Decision To work an extended application analyzing the total manpower of the Department of Defense, visit this text's website at *college.hmco.com*. (*Data Source: U.S. Census Bureau*)

2.7 Exercises

VOCABULARY CHECK: Fill in the blanks.

1. To solve a polynomial inequality, find the ________ numbers of the polynomial, and use these numbers to create ________ ________ for the inequality.
2. The critical numbers of a rational expression are its ________ and its ________ ________.
3. The formula that relates cost, revenue, and profit is ________.

PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at **www.Eduspace.com.**

In Exercises 1–4, determine whether each value of x is a solution of the inequality.

Inequality	*Values*	
1. $x^2 - 3 < 0$	(a) $x = 3$	(b) $x = 0$
	(c) $x = \frac{3}{2}$	(d) $x = -5$
2. $x^2 - x - 12 \geq 0$	(a) $x = 5$	(b) $x = 0$
	(c) $x = -4$	(d) $x = -3$
3. $\dfrac{x+2}{x-4} \geq 3$	(a) $x = 5$	(b) $x = 4$
	(c) $x = -\frac{9}{2}$	(d) $x = \frac{9}{2}$
4. $\dfrac{3x^2}{x^2+4} < 1$	(a) $x = -2$	(b) $x = -1$
	(c) $x = 0$	(d) $x = 3$

In Exercises 5–8, find the critical numbers of the expression.

5. $2x^2 - x - 6$

6. $9x^3 - 25x^2$

7. $2 + \dfrac{3}{x-5}$

8. $\dfrac{x}{x+2} - \dfrac{2}{x-1}$

In Exercises 9–26, solve the inequality and graph the solution on the real number line.

9. $x^2 \leq 9$

10. $x^2 < 36$

11. $(x+2)^2 < 25$

12. $(x-3)^2 \geq 1$

13. $x^2 + 4x + 4 \geq 9$

14. $x^2 - 6x + 9 < 16$

15. $x^2 + x < 6$

16. $x^2 + 2x > 3$

17. $x^2 + 2x - 3 < 0$

18. $x^2 - 4x - 1 > 0$

19. $x^2 + 8x - 5 \geq 0$

20. $-2x^2 + 6x + 15 \leq 0$

21. $x^3 - 3x^2 - x + 3 > 0$

22. $x^3 + 2x^2 - 4x - 8 \leq 0$

23. $x^3 - 2x^2 - 9x - 2 \geq -20$

24. $2x^3 + 13x^2 - 8x - 46 \geq 6$

25. $4x^2 - 4x + 1 \leq 0$

26. $x^2 + 3x + 8 > 0$

In Exercises 27–32, solve the inequality and write the solution set in interval notation.

27. $4x^3 - 6x^2 < 0$

28. $4x^3 - 12x^2 > 0$

29. $x^3 - 4x \geq 0$

30. $2x^3 - x^4 \leq 0$

31. $(x-1)^2(x+2)^3 \geq 0$

32. $x^4(x-3) \leq 0$

Graphical Analysis **In Exercises 33–36, use a graphing utility to graph the equation. Use the graph to approximate the values of x that satisfy each inequality.**

Equation	*Inequalities*	
33. $y = -x^2 + 2x + 3$	(a) $y \leq 0$	(b) $y \geq 3$
34. $y = \frac{1}{2}x^2 - 2x + 1$	(a) $y \leq 0$	(b) $y \geq 7$
35. $y = \frac{1}{8}x^3 - \frac{1}{2}x$	(a) $y \geq 0$	(b) $y \leq 6$
36. $y = x^3 - x^2 - 16x + 16$	(a) $y \leq 0$	(b) $y \geq 36$

In Exercises 37–50, solve the inequality and graph the solution on the real number line.

37. $\dfrac{1}{x} - x > 0$

38. $\dfrac{1}{x} - 4 < 0$

39. $\dfrac{x+6}{x+1} - 2 < 0$

40. $\dfrac{x+12}{x+2} - 3 \geq 0$

41. $\dfrac{3x-5}{x-5} > 4$

42. $\dfrac{5+7x}{1+2x} < 4$

43. $\dfrac{4}{x+5} > \dfrac{1}{2x+3}$

44. $\dfrac{5}{x-6} > \dfrac{3}{x+2}$

45. $\dfrac{1}{x-3} \leq \dfrac{9}{4x+3}$

46. $\dfrac{1}{x} \geq \dfrac{1}{x+3}$

47. $\dfrac{x^2+2x}{x^2-9} \leq 0$

48. $\dfrac{x^2+x-6}{x} \geq 0$

49. $\dfrac{5}{x-1} - \dfrac{2x}{x+1} < 1$

50. $\dfrac{3x}{x-1} \leq \dfrac{x}{x+4} + 3$

Graphical Analysis In Exercises 51–54, use a graphing utility to graph the equation. Use the graph to approximate the values of x that satisfy each inequality.

Equation	Inequalities	
51. $y = \dfrac{3x}{x-2}$	(a) $y \le 0$	(b) $y \ge 6$
52. $y = \dfrac{2(x-2)}{x+1}$	(a) $y \le 0$	(b) $y \ge 8$
53. $y = \dfrac{2x^2}{x^2+4}$	(a) $y \ge 1$	(b) $y \le 2$
54. $y = \dfrac{5x}{x^2+4}$	(a) $y \ge 1$	(b) $y \le 0$

In Exercises 55–60, find the domain of x in the expression. Use a graphing utility to verify your result.

55. $\sqrt{4-x^2}$
56. $\sqrt{x^2-4}$
57. $\sqrt{x^2-7x+12}$
58. $\sqrt{144-9x^2}$
59. $\sqrt{\dfrac{x}{x^2-2x-35}}$
60. $\sqrt{\dfrac{x}{x^2-9}}$

In Exercises 61–66, solve the inequality. (Round your answers to two decimal places.)

61. $0.4x^2 + 5.26 < 10.2$
62. $-1.3x^2 + 3.78 > 2.12$
63. $-0.5x^2 + 12.5x + 1.6 > 0$
64. $1.2x^2 + 4.8x + 3.1 < 5.3$
65. $\dfrac{1}{2.3x - 5.2} > 3.4$
66. $\dfrac{2}{3.1x - 3.7} > 5.8$

67. ***Height of a Projectile*** A projectile is fired straight upward from ground level with an initial velocity of 160 feet per second.
    (a) At what instant will it be back at ground level?
    (b) When will the height exceed 384 feet?

68. ***Height of a Projectile*** A projectile is fired straight upward from ground level with an initial velocity of 128 feet per second.
    (a) At what instant will it be back at ground level?
    (b) When will the height be less than 128 feet?

69. ***Geometry*** A rectangular playing field with a perimeter of 100 meters is to have an area of at least 500 square meters. Within what bounds must the length of the rectangle lie?

70. ***Geometry*** A rectangular parking lot with a perimeter of 440 feet is to have an area of at least 8000 square feet. Within what bounds must the length of the rectangle lie?

71. ***Cost, Revenue, and Profit*** The revenue and cost equations for a product are

$$R = x(75 - 0.0005x) \quad \text{and} \quad C = 30x + 250{,}000$$

where R and C are measured in dollars and x represents the number of units sold. How many units must be sold to obtain a profit of at least \$750,000? What is the price per unit?

72. ***Cost, Revenue, and Profit*** The revenue and cost equations for a product are

$$R = x(50 - 0.0002x) \quad \text{and} \quad C = 12x + 150{,}000$$

where R and C are measured in dollars and x represents the number of units sold. How many units must be sold to obtain a profit of at least \$1,650,000? What is the price per unit?

Model It

73. ***Cable Television*** The percents C of households in the United States that owned a television and had cable from 1980 to 2003 can be modeled by

$$C = 0.0031t^3 - 0.216t^2 + 5.54t + 19.1, \quad 0 \le t \le 23$$

where t is the year, with $t = 0$ corresponding to 1980. (Source: Nielsen Media Research)

(a) Use a graphing utility to graph the equation.

(b) Complete the table to determine the year in which the percent of households that own a television and have cable will exceed 75%.

t	24	26	28	30	32	34
C						

(c) Use the *trace* feature of a graphing utility to verify your answer to part (b).

(d) Complete the table to determine the years during which the percent of households that own a television and have cable will be between 85% and 100%.

t	36	37	38	39	40	41	42	43
C								

(e) Use the *trace* feature of a graphing utility to verify your answer to part (d).

(f) Explain why the model may give values greater than 100% even though such values are not reasonable.